Dr. J's Guide to Set Operations

Union

Definition

The union of set A and B contains all the elements that are in either set. We write the union of A and B as $A \cup B$. Mathematically,

$$A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}.$$

For example,

- $\{1\} \cup \{2\} = \{1, 2\}$
- $\{1, 2\} \cup \{2, 3\} = \{1, 2, 3\}$
- $(0, 1) \cup [1, 2) = (0, 2)$

Intersection

Definition

The intersection of set A and B contains all the elements that are in both sets.
We write the intersection between A and B as $A \cap B$.
Mathematically,

$$A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}.$$

For example,

- $\{1\} \cap \{2\} = \{\,\} = \emptyset$
- $\{1, 2\} \cap \{2, 3\} = \{2\}$
- $(0, 2) \cap [1, 3) = [1, 2)$

Set difference

Definition

The set difference of set A minus B contains all the elements in A that are not in B. We write the set difference of A minus B as $A \setminus B$. Mathematically,

$$A \setminus B = \{\omega \in A : \omega \notin B\}.$$

For example,

- $\{1, 2\} \setminus \{2\} = \{1\}$
- $\{1, 2\} \setminus \{1, 2, 3\} = \emptyset$
- $(0, 10) \setminus [1, 2] = (0, 1) \cup (2, 10)$

Complement

Definition

The complement of set A in S (the universal set: all other sets are subsets this set) contains all the elements in S that are not in A.
We write the complement of A in S as A^C (with S being implied). Mathematically,

$$A^C = \{\omega \in S : \omega \notin A\} = S \setminus A.$$

For example,

- $\{1, 2\}^C$ in $\{1, 2, 3, 4, 5, 6\} = \{3, 4, 5, 6\}$
- $\{1, 2\}^C$ in $\mathbb{N} = \{0, 3, 4, \ldots\}$
- $(0, 10)^C$ in $\mathbb{R}^+ = [10, \infty)$

Properties of set operations

Consider sets A, B, and C that are subsets of the universal set S.

Identity:

- $A \cup \emptyset = A$
- $A \cap S = A$

Commutative property:

- $A \cup B = B \cup A$
- $A \cap B = B \cap A$

Associative property:

- $(A \cup B) \cup C = A \cup (B \cup C)$
- $(A \cap B) \cap C = A \cap (B \cap C)$

Distributive property:

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$
- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Complement:

- $A \cup A^C = S$
- $A \cap A^C = \emptyset$

De Morgan's Laws

De Morgan's Laws relates unions, intersections, and complements.

Suppose $A, B \subseteq S$ where S is the universal set, then the following are true:

$$\begin{aligned} (A \cup B)^C &= A^C \cap B^C \\ (A \cap B)^C &= A^C \cup B^C \end{aligned}$$

Let $S = (0, 10)$, $A = (0, 2)$, and $B = [1, 3)$. Then

- $A^C = [2, 10)$, $\quad B^C = (0, 1) \cup [3, 10)$
  $A^C \cap B^C = [2, 10) \cap ((0, 1) \cup [3, 10))$
  $= ([2, 10) \cap (0, 1)) \cup ([2, 10) \cap [3, 10)) = \emptyset \cup [3, 10) = [3, 10)$, alternatively
- $A \cup B = (0, 3) \implies (A \cup B)^C = [3, 10)$

Set operation summary

- Union: $A \cup B = \{\omega : \omega \in A \text{ or } \omega \in B\}$
- Intersection: $A \cap B = \{\omega : \omega \in A \text{ and } \omega \in B\}$
- Set difference: $A \setminus B = \{\omega \in A : \omega \notin B\}$
- Complement (in S): $A^C = S \setminus A$
- Properties: identity, commutative, associative, distributive, complement
- De Morgan's Laws:

$$\begin{aligned}(A \cup B)^C &= A^C \cap B^C \\ (A \cap B)^C &= A^C \cup B^C\end{aligned}$$