

Dr. J's Guide to Sets

Introduction to Sets

Definition

A **set** is a collection of things or objects. We refer to those objects as **elements** and write a set within brackets $\{ \}$ with elements separated by commas.

For example,

- $\{\text{orange, apple, banana}\}$
- $\{15, 52, 101\}$

Generally, we work with sets of numbers, because we can easily assign a number to other objects, e.g. $15 = \text{orange}$, $52 = \text{apple}$, and $101 = \text{banana}$, and then we can work with a set of numbers.

Large sets

Often, sets are too large for us to write out all the elements, so we'll use shorthand. For example,

- $\{-30, -28, \dots, 48, 50\}$
- $\{4, 5, 6, \dots\}$

Intervals are a type of set

- open interval: $(0, 1)$
- closed interval: $[-2, 3.5]$
- half-open interval: $[-\pi, \pi)$

Elements of a set

To indicate that an element **is in** a set, we use the \in symbol, e.g.

- $20 \in \{2, 4, 6, \dots\}$
- $0 \in (-1, 1)$

To indicate that an element **is not in** a set, we use the \notin symbol, e.g.

- $13 \notin \{2, 4, 6, \dots\}$
- $-\pi \notin (-1, 1)$

Important sets

There are some important sets that we will use in the future:

- empty set: $\emptyset = \{\}$
- natural numbers (including 0): $\mathbb{N} = \{0, 1, 2, \dots\}$
- integers: $\mathbb{Z} = \{\dots, -1, 0, 1, \dots\}$
- real numbers: $\mathbb{R} = (-\infty, \infty)$
- positive real numbers: $\mathbb{R}^+ = (0, \infty)$

Defining sets

The bracket notation can be expanded to include rules using a colon ($:$) which we read as “such that” or “and”,
e.g.

- $(0, 1] = \{\omega \in \mathbb{R} : 0 < \omega \leq 1\}$
- $\{\omega \in \mathbb{N} : \omega < 3\} = \{0, 1, 2\}$
- $\{\omega \in \mathbb{R} : \omega < 3\} = (-\infty, 3)$

Here ω is notation for an element of the set.

Set summary

A set $\{\}$ is composed of elements ω . Elements can either be in \in or not in \notin the set.

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- real numbers: $\mathbb{R} = (-\infty, \infty)$
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