Dr. J's Guide to Subsets & Set Comparisons

Inequalities

Comparing real numbers:

- $1 < 2$
- $100 > 99$
- $\pi \geq \pi$
- $5.5 \leq x < 23 \implies x \in [5.5, 23)$

Inequality rules:

- $x < y \implies x \leq y$
- $x < y \iff y > x$
- $x \leq y, y \leq z \implies x \leq z$
- $x < y, y \leq z \implies x < z$

Comparing sets

How might we compare the following sets?

- empty set: $\emptyset = \{\}$
- $\{1, 2\}$
- $(0, 1)$
- $\mathbb{N} = \{0, 1, 2, \ldots\}$

We will compare two sets by asking these questions:

- Does a set A have all of the elements in a set B?
- If yes, does A have at least one more element than B?

Subsets

Definition

Set B is a subset of a set A if every element of B is an element of A.
If B is a subset of A, we write $B \subseteq A$.

Set B is a proper subset of a set A if

- B is a subset of A and
- A has at least one element that is not in B.

If B is a proper subset of A, we write $B \subset A$.

For example,

- $\{4, 5\} \subseteq \{3, 4, 5\}$, $\{4, 5\} \subset \{3, 4, 5\}$
- $\mathbb{N} \subset \mathbb{Z}$, $\mathbb{Z} \subset \mathbb{R}$
- $\emptyset \subseteq A$ for any set A

Supersets

Definition

Set A is a superset of a set B if every element of B is an element of A. If A is a superset of B, we write $A \supseteq B$.

Set A is a proper superset of a set B if

- A is a superset of B and
- A has at least one element that is not in B.

If A is a proper superset of B, we write $A \supset B$.

For example,

- $\{3, 4, 5\} \supseteq \{4, 5\}$, $\{3, 4, 5\} \supset \{4, 5\}$
- $\mathbb{N} \supset \{0, 1, 2, \ldots, 533\}$
- $\mathbb{R} \supseteq (-\infty, \infty)$

Not subset or superset

Negation: $\nsubseteq$, $\not\subset$, $\nsupseteq$, $\not\supset$

For example,

- $\{1, 1.5, 2\} \nsubseteq \mathbb{N}$
- $\mathbb{N} \nsubseteq \{1, 1.5, 2\}$

CAUTION! Not all inequality rules translate to a set comparison rule.

For example,

- while for $a, b \in \mathbb{R}$, $a \nless b \implies a \geq b$
- $A \not\subset B \not\Longrightarrow A \supseteq B$, e.g. $\{1, 2\} \not\subset \{1, 3\}$ but $\{1, 2\} \nsupseteq \{1, 3\}$.

Set comparison rules

- $A \subset B \implies A \subseteq B$
- $B \supset A \implies B \supseteq A$

- $A \subset B \iff B \supset A$
- $A \subseteq B \iff B \supseteq A$

- $A \subset B, B \subset C \implies A \subset C$
- $A \subseteq B, B \subseteq C \implies A \subseteq C$
- $A \subset B, B \subseteq C \implies A \subset C$

Subset summary

We compare two sets by asking these questions:

- Does a set A have all of the elements in a set B?
- If yes, does A have at least one more element than B?

Set comparisons:

- subset: $B \subseteq A$, proper subset: $B \subset A$
- superset: $A \supseteq B$, proper superset: $A \supset B$

Set comparison rules:

- $A \subset B \iff B \supset A$
- $A \subset B, B \subseteq C \implies A \subset C$