Dr. J's Guide to Partitions

Disjoint

Definition

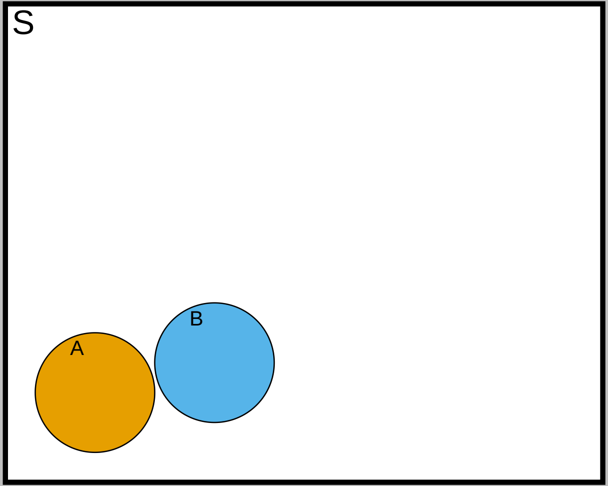
Two sets are disjoint if their intersection is the empty set, i.e. A and B are disjoint if

$$A \cap B = \emptyset.$$

Examples:

- Disjoint:
  - $\{1\}$ and $\{2\}$
  - $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $\{0, 10\}$
  - $(-\infty, 0)$ and $[0, \infty)$
- Not disjoint:
  - $\{1, 2\}$ and $\{2, 3\}$
  - $\mathbb{N}$ and $\mathbb{Z}$
  - $(-\infty, 0]$ and $[0, \infty)$

Disjoint



$$A \cap B = \emptyset$$

Pairwise disjoint

Definition

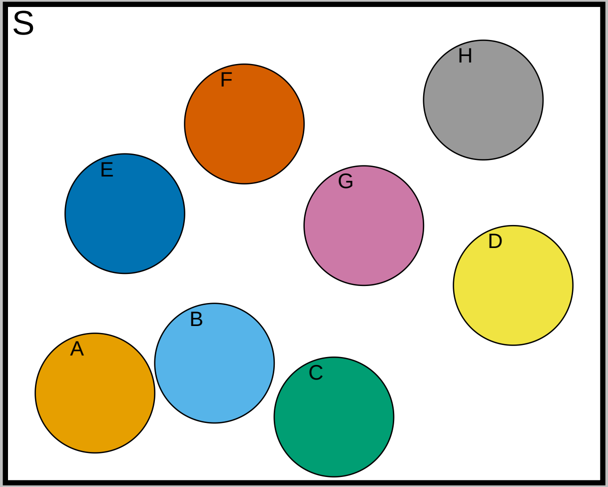
A collection of sets is pairwise disjoint if every pair of sets is disjoint. The collection of sets $A_1, A_2, \ldots$ is pairwise disjoint if

$$A_i \cap A_j = \emptyset \quad \forall \quad i \neq j.$$

For example,

- Pairwise disjoint:
  - $\{\{1,2,3\},\ \{4,5,6\},\ \{7,8,9\},\ \{10,11,12\}\}$
  - $\{\{1\},\ \{2\},\ \{3\}, \ldots\}$
  - $\{(0,1),\ (1,2),\ (2,3), \ldots\}$
- Not pairwise disjoint:
  - $\{\{1,2,3\},\ \{4,5,6\},\ \{7,8,9\},\ \{9,11,12\}\}$
  - $\{\{1,2\},\ \{2,3\},\ \{3,4\},\ \ldots\}$
  - $\{[0,1],\ [1,2],\ [2,3],\ \ldots\}$

Pairwise disjoint



Partition

Definition

A collection of sets is a partition of S if

- none of the sets is the empty set,
- the collection is pairwise disjoint, and
- their union is S.

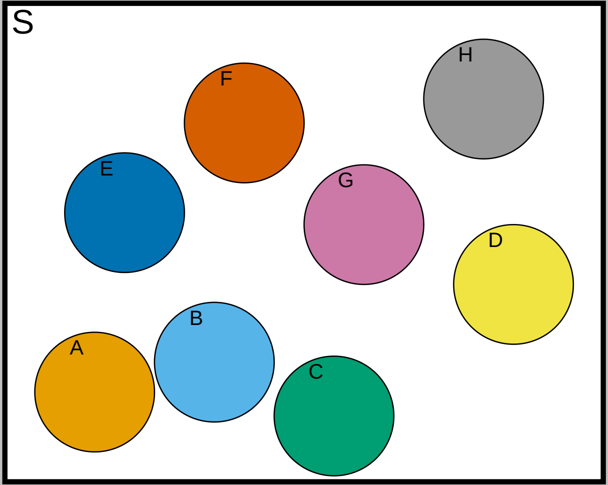
The collection $A_1, A_2, \ldots$ is a partition of S if

- $A_i \neq \emptyset \, \forall \, i$,
- $A_i \cap A_j = \emptyset \quad \forall \quad i \neq j$, and
- $A_1 \cup A_2 \cup A_3 \cup \ldots = S$.

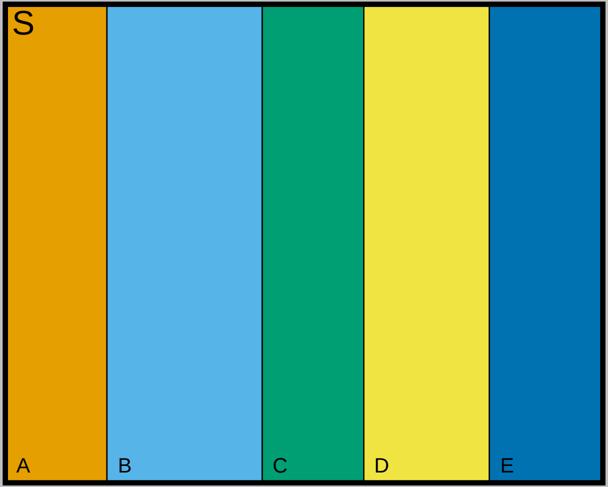
Partition examples

- These are partitions of $\{1, 2, 3, 4, 5, 6\}$:
  - $\{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}\}$
  - $\{\{1, 3, 5\}, \{2, 4, 6\}\}$
  - $\{\{1, 2, 3, 4, 5, 6\}\}$
- These are partitions of $\mathbb{N}$:
  - $\{\{0\}, \{1\}, \{2\}, \ldots\}$
  - $\{\{0, 2, 4, \ldots\}, \{1, 3, 5, \ldots\}\}$
  - $\{\{0, 1\}, \{2, 3\}, \{4, 5\}, \ldots\}$
- These are partitions of $\mathbb{R}^+$:
  - $\{(0, 1], (1, 2], (2, 3], \ldots\}$
  - $\{(0, 1), \{1\}, (1, 2), \{2\}, (2, 3), \{3\}, \ldots\}$

Not a partition

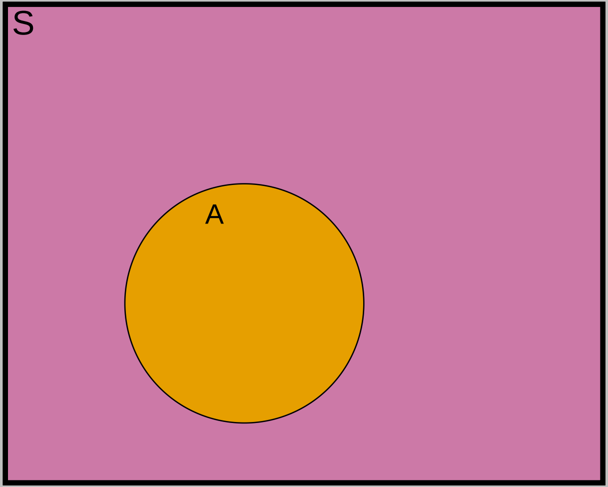


Partition: rectangles



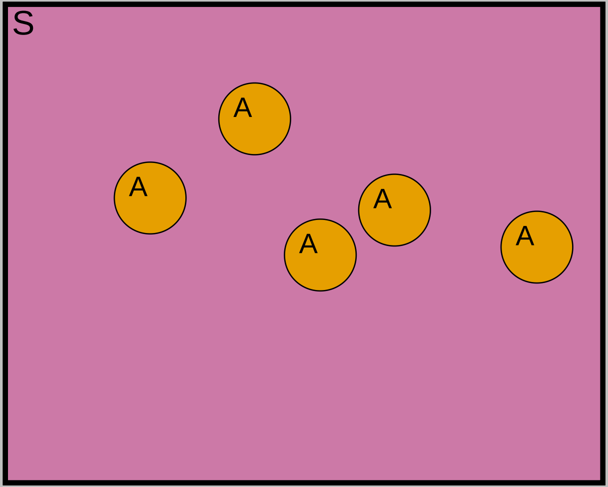
$$\{A, B, C, D, E\}$$

Partition of S: A subset of S and its complement



$$\left\{A, A^C\right\}$$

Partition of S: A subset of S and its complement



$$\left\{A, A^C\right\}$$

Partition Summary

- Disjoint: $A \cap B = \emptyset$
- Pairwise disjoint: $A_i \cap A_j = \emptyset \quad \forall \quad i \neq j$
- Partition of S:
  - $A_i \neq \emptyset \quad \forall \quad i$
  - $A_i \cap A_j = \emptyset \quad \forall \quad i \neq j$
  - $\cup_{i=1}^{\infty} A_i = S$