Dr. J's Guide to Cardinality

Cardinality

How many elements are in these sets?

- $A = \{1, 2, \ldots, 100\}$
- all subsets of A
- $\mathbb{N}$
- $(0, 1)$
- $\mathbb{R}$

Definition

The cardinality of a set A, written $|A|$, is the number of elements in the set.

We will discuss identifying whether a set has finite, countably infinite, or uncountably infinite cardinality.

Finite

Definition

A set is finite if its cardinality less than the cardinality of $\mathbb{N}$.

Finite sets:

- cards in a standard deck of cards
- U.S. social security numbers
- 32-digit passwords using letters, numbers, and symbols
- grains of sand on a beach

Countably infinite

Definition

A set is countably infinite (or countable) if its cardinality is the same as the cardinality of $\mathbb{N}$.

Countable sets:

- positive integers: $\{1_0, 2_1, 3_2, \ldots\}$
- $\mathbb{Z}$:

$$\begin{array}{ccccc} 0_0, & 1_1, & 2_3, & 3_5, \ldots \\ & -1_2, & -2_4, & -3_6, \ldots \end{array}$$

- possibilities for the number of tails before flipping a head: $\mathbb{N}$

Uncountably infinite

Definition

A set is uncountably infinite (or uncountable) if its cardinality is greater than the cardinality of $\mathbb{N}$.

Uncountable sets:

- $\mathbb{R}$
- $\mathbb{R}^+$
- $(0, 1)$
- reaction times: $\mathbb{R}^+$

Summary

- Cardinality
  - Finite ($< |\mathbb{N}|$) (Discrete)
  - Countably infinite ($= |\mathbb{N}|$) (Discrete)
  - Uncountably infinite ($> |\mathbb{N}|$) (Continuous)