Dr. J's Guide to Probability

Experiments and Events

Definition

An experiment is any data collection process. The result of any particular experiment is the outcome of that experiment. The set of all possible outcomes is the sample space, denoted S. Subsets of the sample space are events.

Six-sided die rolling experiment:

- sample space: $\{1, 2, 3, 4, 5, 6\}$
- events: odds ($\{1, 3, 5\}$) and evens ($\{2, 4, 6\}$)

Customer satisfaction survey:

- sample space: {Very unsatisfied, Unsatisfied, Neutral, Satisfied, Very satisfied}
- event: satisfied or better ({Satisfied, Very satisfied})

Probabilities 6-sided die rolling

For a 6-sided die roll, a reasonable belief is

$$P(\{i\}) = \frac{1}{6} \quad \text{for} \quad i = 1, 2, 3, 4, 5, 6.$$

For non-singleton events, we will calculate the probabilities like this

$$P(A) = \sum_{\omega \in A} P(\{\omega\}).$$

For example,

$$P(\{1, 3, 5\}) = P(\{1\}) + P(\{3\}) + P(\{5\}) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}.$$

Kolmogorov's axioms of probability

Definition

Let A be an event from an experiment with sample space S. Then, a probability, $P(A)$, has the following properties:

1. $P(A) \geq 0$ for $A \subseteq S$
2. $P(S) = 1$
3. Given pairwise disjoint events $A_1, A_2, \ldots \subseteq S$,

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

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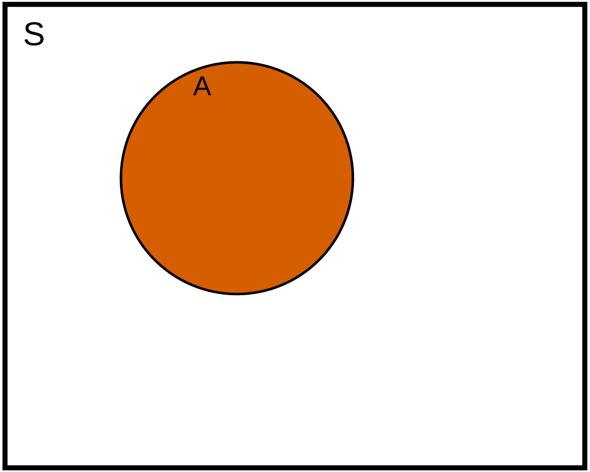
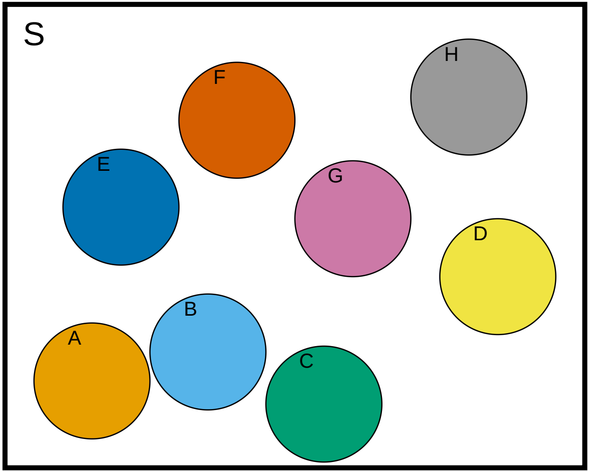
Visualizing probability



Visualizing Kolmogorov's 3rd axiom



Probability: roll a 6-sided die

6-sided die rolling probabilities:

$$P(\{i\}) = \frac{1}{6}, \quad i = 1, \ldots, 6, \qquad\qquad P(A) = \sum_{\omega \in A} P(\{\omega\}).$$

These satisfy Kolmogorov's axioms because

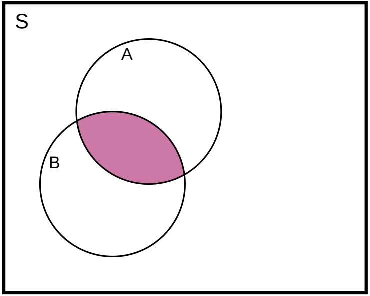
1. All events have $P(A) \geq 0$.
2. $P(S) = P(\{1, 2, 3, 4, 5, 6\}) = P(\{1\}) + P(\{2\}) + P(\{3\}) + P(\{4\}) + P(\{5\}) + P(\{6\}) = 1$.
3. For pairwise disjoint $A_1, A_2, \ldots$,

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = P(A_1 \cup A_2 \cup \cdots) = \sum_{\omega \in A_1 \cup A_2 \cup \cdots} P(\{\omega\}) = \sum_{i=1}^{\infty} \sum_{\omega \in A_i} P(\{\omega\}) = \sum_{i=1}^{\infty} P(A_i).$$

Consequences of Kolmogorov's axioms

From Kolmogorov's axioms, we can prove the following

- $P(\emptyset) = 0$
- monotonicity: for $A \subseteq B$, $P(A) \leq P(B)$
- $0 \leq P(A) \leq 1$.
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$:



Conditional probability

Definition

The conditional probability of a set B given a set A is

$$P(B|A) = \frac{P(B \cap A)}{P(A)}, \quad P(A) > 0$$

where

- $P(B|A)$ is the conditional probability of B given A,
- $P(B \cap A)$ is the joint probability of B and A, and
- $P(A)$ as the marginal probability of A.

Visualizing conditional probability

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Error in grid.draw(plot):  object 'partition' not found
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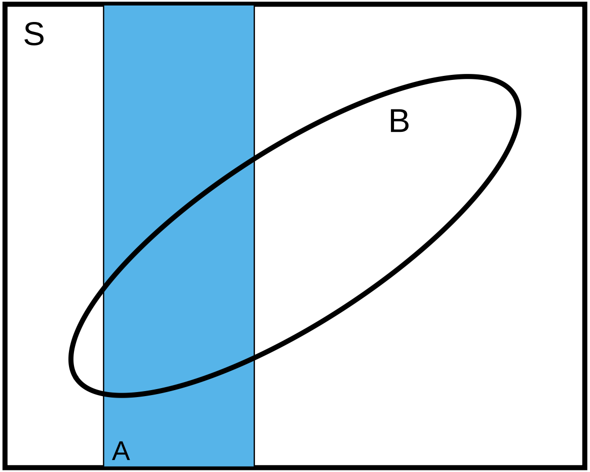

Conditional probability: die rolling

Consider rolling a 6-sided die we want to calculate the probability of the roll being less than 3 when we know the roll was odd.

Let

- $B = \{1, 2\}$
- $A = \{1, 3, 5\}$

and thus $B \cap A = \{1\}$,
then

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(\{1\})}{P(\{1,3,5\})}$$

$$= \frac{1/6}{3/6} = \frac{1}{3}$$

Summary

- Probability
  - Conditional
  - Joint
  - Marginal