Dr. J's Guide to Permutations (without replacement)

Fundamental Rule of Counting

Definition

The Fundamental Rule of Counting states that if there are a ways of doing something **and** b ways of doing something else, then there are $a \times b$ ways of doing those two things.

Examples:

- How many ways are there to draw two cards out of a standard deck of cards?

$$52 \times 51 = 2,652$$

- How many Powerball lottery draws are there?

$$\underbrace{69 \times 68 \times 67 \times 66 \times 65}_{\text{white}} \times \underbrace{26}_{\text{red}} = 35 \times 10^{11}$$

Intuition behind the formula for permutations

Definition

A permutation (without replacement) is an ordered set of k elements taken from a set of n elements where elements cannot be repeated.

How many choices do we have for the

- first item? n
- second item? $n-1$
- $\vdots$
- kth item? $n-k+1$

By the Fundamental Rule of Counting, we have

$$n \times (n-1) \times \cdots \times (n-k+1) = \frac{n!}{(n-k)!}$$

of ways to choose these items.

Cross-over clinical trial

In a cross-over clinical trial, patients are given a series of treatments with no treatments being repeated. How many possibly treatment plans are there is there are 3 treatments and each patient receives 2 unique treatments?

This is a permutation without replacement with $n = 3$ and $k = 2$. Thus, there are

$$\frac{3!}{(3-2)!} = 6$$

treatment plans. If the treatments are labeled $A, B,$ and C, the plans are

AB AC BA BC CA CB

Shuffling a deck of cards

How many possible rearrangements are there for a standard 52-card deck?

There are $n = 52$ cards and there are $k = 52$ in the final deck. Thus, there are

$$\frac{52!}{(52-52)!} = \frac{52!}{0!} = 52! \approx 10^{67.9}$$

where $0! = 1$.

This is the *standard* permutation.

Summary

- Fundamental Rule of Counting
- Intuition behind permutation formula: $\frac{n!}{(n-k)!}$
- Examples:
  - Cross-over trials
  - Shuffling cards