

Dr. J's  
Guide to  
Combinations  
(without replacement)

# Intuition behind the formula for combinations

## Definition

A **combination (without replacement)** is an **unordered set** of  $k$  elements taken from a set of  $n$  elements where elements **cannot be repeated**.

- Same as permutation (without replacement) but without ordering.
- Calculate  $\frac{n!}{(n-k)!}$  and divide by  $k!$ , permutations of  $k$  items.

Thus for combinations (without replacement), there are

$$\frac{n!}{(n-k)!} \div k! = \underbrace{\frac{n!}{(n-k)!k!}}_{\text{by definition}} = \binom{n}{k}$$

combinations where  $\binom{n}{k}$  is read “ $n$  choose  $k$ ”.

# Texas hold 'em

How many possible starting hands are there in Texas hold 'em? (Two cards)

This is a combination (without replacement) from a deck of size  $n = 52$  to a hand of size  $k = 2$ .

Thus, there are

$$\binom{52}{2} = \frac{52!}{(52 - 2)!2!} = 1,326$$

possible starting hands in Texas Hold 'Em.



# Euchre

Euchre is a card game played with a reduced deck: 9, 10, Jack, Queen, King, and Ace in each suit.

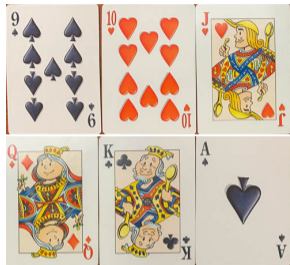
Each player is dealt 5 cards.

How many unique starting hands are there in Euchre?

This is a combination (without replacement) from a deck of size  $n = 6 \times 4 = 24$  to a hand of size  $k = 5$ . Thus, there are

$$\binom{24}{5} = \frac{24!}{(24 - 5)!5!} = 42,504$$

different starting hands in Euchre.



# Summary

- Intuition behind combination formula:

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

- Card examples:
  - Texas Hold 'Em
  - Euchre